Spatial independence of wind storm gust extremes

David Stephenson and Laura Dawkins
Exeter Climate Systems
Geostatistical modelling of windstorms

**Concept:**
Model wind gust speed footprints as realisations of a spatial stochastic process.

This alternative to more mechanistic models based on tracks provides a fast and useful benchmark for testing traditional cat models. *(i.e. scrambled rather than fried eggs!)*


Definition of windstorm footprint

Maximum 3-second wind-gust speed at each grid point in a 72 hour period covering the passage of the storm, centred on the time at which the maximum wind speed over land occurs.

Example: Footprint for windstorm Daria (24th - 26th January 1990)

Large sample of 6301 storms identified in extended winters (October-March) using objective tracking algorithm.

Then footprints created for each storm by dynamically downscaling ERA-Interim using the Met Office 25km resolution North Atlantic-European operational NWP model.

High dimensional data ~15000 grid points!
How good are the model wind gust speeds?

→ Good agreement but with model speeds overestimating at low speeds and underestimating at high speeds (RMSE of 4.5m/s for top 50)
Aggregate losses depend on joint distribution

Joint loss (points in a) is strongly determined by dependency between sites: e.g. some dependency between London & Amsterdam but not London & Madrid
Measuring extremal dependence in wind speeds

\[ \chi(p) = \frac{\Pr(X > x_{1-p} \& Y > y_{1-p})}{\Pr(X > x_{1-p})} \]

\[ \bar{\chi}(p) = \frac{2 \log p}{\log \chi p} - 1 \]

As \( p = \Pr(X > x_{1-p}) \to 0 \) either:

\( \chi \to 0 \) (extremal independence)
or \( \neq 0 \) and \( \bar{\chi} \to 1 \) (extremal dependence).

Perfect independence \( (\chi = p, \bar{\chi} = 0) \)
Perfect dependence \( (\chi = 1, \bar{\chi} = 1) \)

\( \chi \to 0 \) so no evidence of extremal dependence as assumed in previous studies

Copula models of dependency

Dependency between two variables can be quantified by factoring out the marginal distributions.

Any joint probability can be written as a function of the marginal probabilities:

Pr(X ≤ x & Y ≤ y) = C(u, v)

where

U = Pr(X ≤ x) = F_X(x)

V = Pr(Y ≤ y) = F_Y(y)

\[ C(u, v) = e^{-((-\ln u)^r + (-\ln v)^r)^{1/r}} \]

\[ C(u, v) = \Phi_{\Sigma}(\Phi^{-1}(u), \Phi^{-1}(v)) \]

\[ C(u, v) = \min(u, v) \]
Wind speed $p=0.01$ dependency with London

$\chi$

(a) (b)

Empirical copula

Gaussian copula

(c) (d)

Gumbel copula

$\rightarrow$ Gaussian copula matches empirical whereas Gumbel overestimates dependence
Dependency between different locations across Europe

\[ \bar{\chi} \rightarrow 2\eta - 1 \]

→ Very little evidence for asymptotic extremal dependence
Distribution of total loss given a $p<0.01$ loss in London

Gaussian copula matches empirical whereas Gumbel overestimates total losses
Extremal dependence in homogeneous turbulence

Homogeneous turbulence can be simulated using spatially correlated Gaussian velocity components (Von Karman, 1937).

What does this imply for extremal dependence of squared wind speeds $u_1^2 + v_1^2$ and $u_2^2 + v_2^2$ at different locations?

![Scatter plot and cumulative distribution function](image)

Even for highly correlated velocities, the joint probability of exceedance appears to drop off faster than the marginal probability at higher thresholds i.e. $\chi \to 0$ (not asymptotically extremally dependent).
Summary

• There is not strong evidence for spatial extreme dependence in 6301 dynamically-downscaled European windstorm footprints;

• Dependence and aggregate losses overestimated by copula models that have extremal dependence e.g. Gumbel;

• Gaussian copula appears to fit the data well, which allows fast geostatistical simulation of footprints.

Thank you for your attention:
d.b.stephenson@exeter.ac.uk